Elementary Algebra Study Guide

Some Basic Facts

This section will cover the following topics

Notation

Order of Operations

Notation

Math is a language of its own. It has vocabulary and punctuation (notation) just like any other language. To help you get ready for the placement exam, here is a list of some important notation to know.

<table>
<tr><th>Notation</th><th>Description</th></tr>
<tr><td>Multiplication</td><td>
Multiplication can be expressed by the symbols $\times$, $\cdot$, $*$, or ( ).

Examples
<table>
<tr><td>$\times$</td><td>$5 \times 2 = 10$</td></tr>
<tr><td>$\cdot$</td><td>$5 \cdot 2 = 10$</td></tr>
<tr><td>$*$</td><td>$5 * 2 = 10$</td></tr>
<tr><td>( )</td><td>$5(2) = 10$</td></tr>
</table>
</td></tr>
<tr><td>Division</td><td>
Division can be expressed by the symbols $\div$, /, $-$, or $\overline{)\quad}$.

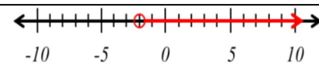
Examples
<table>
<tr><td>$\div$</td><td>$10 \div 2 = 5$</td><td rowspan="3">$\begin{array}{r} 5 \\ 2\overline{)\,10} \\ -\underline{10} \\ 0 \end{array}$</td></tr>
<tr><td>/</td><td>$10/2 = 5$</td></tr>
<tr><td>$\frac{a}{b}$</td><td>$\frac{10}{2} = 5$</td></tr>
</table>
</td></tr>
<tr><td>Exponents</td><td>
Exponents is a shortcut for multiplication. For example, 3^4 is a shortcut way of saying, “multiply 3 by itself 4 times.” In other words,

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$
</td></tr>
<tr><td>Square Roots</td><td>
The square root symbol is $\sqrt{\ }$. It means the number that, when multiplied by itself, results in the value inside the root. For example, $\sqrt{4} = 2$ because $2 \times 2 = 4$

Common Roots
<table>
<tr><td>$\sqrt{1} = 1$</td><td>$\sqrt{4} = 2$</td><td>$\sqrt{9} = 3$</td><td>$\sqrt{16} = 4$</td><td>$\sqrt{25} = 5$</td><td>$\sqrt{36} = 6$</td></tr>
<tr><td>$\sqrt{49} = 7$</td><td>$\sqrt{64} = 8$</td><td>$\sqrt{81} = 9$</td><td>$\sqrt{100} = 10$</td><td>$\sqrt{121} = 11$</td><td>$\sqrt{144} = 12$</td></tr>
</table>
</td></tr>
</table>

Notation	Description
Inequalities and interval notation	

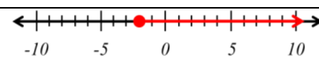
Greater Than

Inequality	Interval Notation	Number Line
$x > -2$	$(-2, \infty)$	

Note: the "$>$" in > -2 , the "(" in $(-2, \infty)$, and the hollow "○" mean include every number bigger than -2 *BUT NOT* -2 itself.

For example, driving strictly faster than the speed limit may get you a ticket, but driving at the speed limit will not.

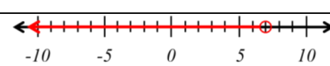
Greater Than or Equal To

Inequality	Interval Notation	Number Line
$x \geq -2$	$[-2, \infty)$	

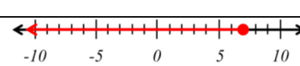
Note: the "≥" in $x \geq -2$, the "[" in $[-2, \infty)$, and the solid "●" mean include every number bigger than -2 *AND* -2 itself.

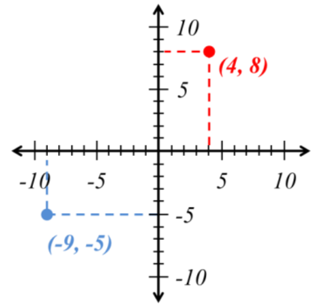
The standard legal age for getting a driver's license is 16 or older. The age of 16 is included.

Less Than

Inequality	Interval Notation	Number Line
$x < 7$	$(-\infty, 7)$	

Less Than or Equal To

Inequality	Interval Notation	Number Line
$x \leq 7$	$(-\infty, 7]$	

Notation	Description
Graphing Points in a Plane	Points are written in the form of (x, y), which is called an “ordered pair.” The x represents the left-right distance from the center of the plane, while the y represents the up-down distance from the center 

Order of Operations

<table>
<tr><th colspan="2">PEMdAs</th><th>Description</th></tr>
<tr><td>1</td><td>P = Parentheses</td><td>In this case, the term “parentheses” will include anything that would be considered a grouping symbol such as [ ], which are called square brackets.

Also included in this list is the fraction line.

For example, in the fraction $\frac{5+3}{2+7}$ first focus on the numerator and denominator separately to get $\frac{5+3}{2+7} = \frac{8}{9}$</td></tr>
<tr><td>2</td><td>E = Exponents</td><td>Exponents are a shortcut for multiplication.

2^4 means multiply 2 by itself 4 times.</td></tr>
<tr><td>3</td><td>M = Multiplication

And

D = Division</td><td>Multiplication and Division are considered the same in the order of operations. It is important to note, however, that they should be done left to right.
<table>
<tr><th>Example 1</th><th>Example 2</th></tr>
<tr><td>$10 \div 2 \times 3 = 5 \times 3 = 15$</td><td>$6 \times 3 \div 2 = 18 \div 2 = 9$</td></tr>
</table></td></tr>
<tr><td>4</td><td>A = Addition

And

S = Subtraction</td><td>As with multiplication and division, Addition and Subtraction are considered the same. They are also approached left to right.
<table>
<tr><th>Example 1</th><th>Example 2</th></tr>
<tr><td>$10 - 2 + 3 = 8 + 3 = 11$</td><td>$6 + 3 - 2 = 9 - 2 = 7$</td></tr>
</table></td></tr>
</table>

Practice Problems

1. $3 + 4 \cdot 5 - 6$	2. $3^2 + 4 * 5 - 6 \div 2$
3. $3^2 + 4 * (6 - 2) - 5$	4. $\frac{3+5*2}{4*6-9}$

Answers

1. 17	2. 26
3. 20	4. $\frac{13}{15}$

Additional Help

Hippocampus has a video on the Order of Operations at the following link

Order of Operations

You can also search YouTube.com for "order of operations"

Fractions

This section will cover the following topics

Reducing to Lowest Terms

Converting between Mixed Numbers and Improper Fractions

Multiplying and Dividing

Adding and Subtracting

Reducing

Fractions have the ability to look different without changing value. A common example that is given is slicing a pizza. Suppose someone was feeling very hungry and wanted a very large slice of pizza so they cut the pizza into two slices by cutting right down the middle. Eating only one of those very large slices would mean eating exactly ½ of the pizza. But if the pizza were bought from a shop where it was already sliced into eight pieces, that person could easily still eat ½ of the pizza by eating 4 out of 8 slices. In fractions, that means $\frac{4}{8} = \frac{1}{2}$. Even though these fractions look quite different, they still represent the same value. Here are some examples to illustrate how fractions are reduced, and what it means to be reduced to **lowest terms**. The key is to look for common divisors.

Example 1

$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$	In looking at the fraction $^6/_9$, we are looking for a number which will divide both 6 and 9, which is **3**. 3 is called a common divisor. $^2/_3$ is in lowest terms because 2 and 3 do not share a common divisor, so we are done.

Example 2

$\frac{24}{30} = \frac{24 \div \color{red}{6}}{30 \div \color{red}{6}} = \frac{4}{5}$	$^{24}/_{30}$ is a bit more challenging because there are many numbers which divide both 24 and 30. They are 2, 3, and 6. We will use **6** to do the reduction because it is the greatest; called the greatest common divisor.

Example 3

$\frac{216}{240} = \frac{216 \div \color{red}{2}}{240 \div \color{red}{2}} = \frac{108 \div \color{green}{2}}{120 \div \color{green}{2}}$ $= \frac{54 \div \color{green}{2}}{60 \div \color{green}{2}} = \frac{27 \div \color{cyan}{3}}{30 \div \color{cyan}{3}} = \frac{9}{10}$	$^{216}/_{240}$ looks nearly impossible, but will turn out to be much easier than it looks if we take it step-by-step. That is, let's start with dividing both 216 and 240 by **2** since they are both even numbers. From there we will continue by dividing by **2**, **2**, and finally **3**.

A Quick Tip

Whenever doing math, either in a class or on the placement exam, fractions are always reduced.

Converting between Mixed Numbers and Improper Fractions

Let's start by giving an example of a mixed number and an improper fraction. $3\frac{1}{2}$ is a **mixed number** because is mixes a whole number (the 3) with a fraction (the $\frac{1}{2}$). The fraction $\frac{7}{2}$ is an **improper fraction** because the numerator is larger than the denominator. We can convert between these two forms in the following ways.

Convert a Mixed Number to an Improper Fraction: $3\frac{1}{2}$ to $\frac{7}{2}$

First multiply the whole **3** by the denominator **2**.	$\color{red}{3}\frac{1}{\color{red}{2}}$: $\color{red}{3} \times \color{red}{2} = 6$
Then take the resulting 6 and add the numerator of **1.**	$3\frac{\color{green}{1}}{2}$: $6 + \color{green}{1} = 7$
The 7 become the new numerator and the 2 remains the denominator.	$\frac{7}{2}$

Convert an Improper Fraction to a Mixed Number: $\frac{7}{2}$ to $3\frac{1}{2}$

First we must determine how many times **2** goes into 7 without going over 7. The answer (highlighted in yellow) is **3** times with **1** left over.	$\frac{7}{2}$:	$2 \times 1 = 2$ $2 \times 2 = 4$ $2 \times 3 = 6$ $2 \times 4 = 8$
	$\frac{7}{2} = 3\frac{1}{2}$:	

Multiplying and Dividing

Multiplication and Division are relatively straightforward. For multiplication remember to multiply across (numerator × numerator and denominator × denominator) and then reduce. For division of fractions, simply change division to multiplication. To change division into multiplication, remember Copy – Dot – Flip. That is, Copy the first fraction – change division (÷) to multiplication (·) – and flip the section fraction ($\frac{a}{b} \to \frac{b}{a}$).

Multiplication Example 1: $\frac{4}{5} \cdot \frac{3}{8}$

$\frac{4}{5} \cdot \frac{3}{8} = \frac{12}{40}$	Multiply Across
$\frac{12 \div 4}{40 \div 4} = \frac{3}{10}$	Reduce to Lowest Terms

Multiplication Example 2: $\frac{10}{6} \cdot \frac{9}{12}$

$\frac{10}{6} \cdot \frac{9}{12} = \frac{90}{72}$	Multiply Across
$\frac{90 \div 18}{72 \div 18} = \frac{5}{4}$	Reduce to Lowest Terms

Division Example 1: $\frac{5}{6} \div \frac{10}{7}$

$\frac{5}{6} \div \frac{10}{7} = \frac{5}{6} \cdot \frac{7}{10}$	**Copy – Dot – Flip**
$\frac{5}{6} \cdot \frac{7}{10} = \frac{35}{60}$	Multiply Across
$\frac{35 \div 5}{60 \div 5} = \frac{7}{12}$	Reduce to Lowest Terms

Division Example 2: $\frac{8}{9} \div \frac{6}{5}$

$\frac{8}{9} \div \frac{6}{5} = \frac{8}{9} \cdot \frac{5}{6}$	**Copy – Dot – Flip**
$\frac{8}{9} \cdot \frac{5}{6} = \frac{40}{54}$	Multiply Across
$\frac{40 \div 2}{54 \div 2} = \frac{20}{27}$	Reduce to Lowest Terms

Adding and Subtracting

While multiplying and dividing fractions are straightforward, adding and subtracting fractions are not – UNLESS – you have a common denominator between the two fractions. So that's the trick – get a common denominator before adding/subtracting.

Addition Example: $\frac{2}{3} + \frac{1}{4}$

$\frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$ $\frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}$	The common denominator is **12.** Multiply first fraction by **4**'s and the second by **3**'s
$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$	Add the numerators and reduce if necessary

Subtraction: $\frac{5}{6} - \frac{2}{9}$

$\frac{5 \cdot 3}{6 \cdot 3} = \frac{15}{18}$ $\frac{2 \cdot 2}{9 \cdot 2} = \frac{4}{18}$	The common denominator is **18.** Multiply first fraction by **3**'s and the second by **2**'s
$\frac{15}{18} - \frac{4}{18} = \frac{11}{18}$	Subtract the numerators and reduce if necessary

A Quick Tip

It is easy to confuse the techniques between multiplying/dividing fractions and adding/subtracting fractions, and it often is related to when a common denominator is needed. Remember that a common denominator is needed only for addition and subtraction of fractions.

Practice Problems

Reduce the following fractions to lowest terms

1. $\frac{12}{15}$
2. $\frac{32}{40}$
3. $\frac{12}{16}$

Convert the following mixed numbers to improper fractions, and improper fractions to mixed numbers.

4. $2\frac{1}{3}$
5. $5\frac{3}{7}$
6. $\frac{22}{3}$
7. $\frac{19}{8}$

Perform the following multiplication/division problems with fractions.

8. $\frac{2}{5} \cdot \frac{10}{3}$
9. $\frac{6}{9} \cdot \frac{3}{8}$
10. $\frac{3}{4} \div \frac{1}{2}$
11. $\frac{2}{7} \div \frac{5}{14}$

Perform the following addition/subtraction problems with fractions.

12. $\frac{1}{4} + \frac{3}{5}$
13. $\frac{4}{9} + \frac{2}{6}$
14. $\frac{5}{6} - \frac{3}{4}$
15. $\frac{4}{5} - \frac{2}{3}$

Answers

Reduce the following fractions to lowest terms

1. $\frac{4}{5}$	2. $\frac{4}{5}$	3. $\frac{3}{4}$

4. $\frac{7}{3}$	5. $\frac{38}{7}$	6. $7\frac{1}{3}$	7. $2\frac{3}{8}$
8. $\frac{4}{3}$	9. $\frac{1}{4}$	10. $\frac{3}{2}$	11. $\frac{4}{5}$
12. $\frac{17}{20}$	13. $\frac{7}{9}$	14. $\frac{1}{12}$	15. $\frac{2}{15}$

Additional Help

Hippocampus has videos on fractions at the following links: Simplifying Fractions; Proper and Improper Fractions; Multiplying Fractions; Dividing Fractions; Adding Fractions; Subtracting Fractions

You can also search YouTube.com for “reducing fractions”, “converting fractions”, or “adding/subtracting/multiplying/dividing fractions”

Substitution

This section will cover the following topics

Definition of Expression and Equation

Evaluating an Expression Using Substitution

Checking the Solution to an Equation Using Substitution

Definition of Expression and Equation

Both expressions and equations combine numbers, variables (such as x and y), and arithmetic operations (such as $+$, $-$, $\times$, and $\div$). **The difference between expressions and equations is equations have an equals sign and expressions do not.** Below are some examples of each kind.

Expressions	Equations
$2x$	$2x = 6$
$2x + 3y$	$2x + 3y = 7$
$3xy^2 + 2x$	$3xy^2 + 2x + 5 = 2x - 4$

Evaluating an Expression Using Substitution

The word "evaluate" means to find the numerical value of an expression and it requires that you know the value of all variables. For example, the expression 2x cannot be evaluated unless we know what number x is equal to. Let's say, for example, that we did know $x = 3$. We could then use substitution to find the value of 2x by simply replacing every x with 3. That is,

$2x - 2(3)$ or $2x = 6$.

More Examples

Example 1

Expression	$2x + 3y$
Value of the Variables	$x = -1, y = 3$
Substitution and Evaluation	$2(-1) + 3(3) =$ $-2 + 9 = 7$

Example 2

Expression	$3xy^2 + 2x$
Value of the Variables	$x = 1, y = 3$
Substitution and Evaluation	$3(1)(3)^2 + 2(1)$ $27 + 2 = 29$

A Quick Tip

Substitution is a very useful tool when taking a placement exam. You can expect to see a few of these on the exam, and with just a bit of practice you can increase your placement score.

Checking the Solution to an Equation Using Substitution

The idea of using substitution with equations is the same as using substitution on expressions with one exception; both side of the equation must be equal (the same number). For example, if we have the equation 2x $= 6$ and we substitute $x = 3$, then get $2(3) = 6$ or $6 = 6$. This is a true statement; 6 does equal 6. We would say that the solution $x = 3$ "checks".

But let's say we instead have the equation 2x $= 6$ and we substitute $x = 4$. We get $2(4) = 6$ or $8 = 6$. This is a false statement; 8 does not equal 6, and we say that the solution $x = 4$ "does not check".

More Examples

Example 1

Equation	$2x + 3y = 7$
Value of the Variables	$x = -1, y = 3$
Substitution and Evaluation	$2(-1) + 3(3) = 7$ $-2 + 9 = 7$

Example 2

Equation	$3xy^2 + 2x = 5$
Value of the Variables	$x = 1, y = 3$
Substitution and Evaluation	$3(1)(3)^2 + 2(1) = 5$ $27 + 2 = 5$

	$7 = 7$ *The solution checks*		$29 = 5$ *The solution does not check*

A Quick Tip

If you get an equation to solve on the placement exam, either solve it directly or check each possible answer using substitution as an alternative strategy. Choose the strategy that gives you the best chance to succeed!

Practice Problems

Evaluate the following expressions or check the equation using substitution

1. $a + 2b - 3c$ $a = 1, b = -2, c = 3$	2. $3xy^2z^3$ $x = \frac{1}{2}, y = 3, z = 2$	3. $x^2 - 2x + 7$ $x = -4$
4. $3x + 1 = 4$ $x = 1$	5. $3x - 2 = 4x^2$ $x = -2$	6. $x^2 + 3x - 1 = -3$ $x = -1$

Answers

1. *–12*	2. *108*	3. *31*
4. *The solution checks*	5. *The solution does not check*	6. *The solution checks*

Linear Equations

This section will cover the following topics

What is a Linear Equation?

Solving One-Step Linear Equations

Solving Two-Step Linear Equations

Solving Linear Equations That Include Parentheses

What is a Linear Equation?

A linear equation is an equation in which the variable (or variables) have an exponent of *1*, and in which the variables to do not appear in the denominator. Here are some examples of equations that are both linear and non-linear

Equation	Linear (Yes or No?)
$x + 7 = 5$	**Yes –** the x does not appear to have an exponent, but in fact there is an implied exponent of *1*. That is, when we write x, we mean x^1.
$5x + 7y = 10$	**Yes –** Even though there are multiple variables, they each have an implied exponent of *1*.

$x^2 + 7 = 5$	**No –** In this case, the exponent of the x is 2. Thus, the equation is not a linear equation.
$\frac{1}{x} + 7x = 5$	**No –** Although the x variable has no visible exponent, it is in the denominator of the first term. Thus, the equation is not a linear.

Solving One-Step Linear Equations

A key thing to keep in mind is that solving an equation means to isolate the variable on one side of the equals sign. That is, the end result should look like, "x = _________". A One-Step Linear Equation is a linear equation that is a single operation away from being solved; either through addition, subtraction, multiplication, or division. Here are some examples

Example 1

$x + 7 = 5$	In this example, the only step that needs to be done is to eliminate the +7 from the left hand side of the equation.
$x + 7 - 7 = 5 - 7$	This is done by the "$- 7$" – remember that what is done to one of the equation must be done to both.
$x = -2$	

Example 2

$2x = 10$	In this example, the only step that needs to be done is to eliminate the **2** in front of the x.
$\frac{\cancel{2}x}{\cancel{2}} = \frac{10}{2}$	This is done by dividing both sides of the equation by **2**.
$x = 5$	

Additional Help

Hippocampus has a video on Solving Linear Equations at the following link:

Solving One Step Linear Equations

You can also search YouTube.com for "solving linear equations"

Solving Two-Step Linear Equations

Solving Two-Step Linear Equations puts together the pieces in the above examples to solve a single problem. In other words, we will (1) add/subtract and then (2) multiply/divide. It will be done in that order, too.

Example 1

$2x + 7 = 5$	In this example, the two steps that will need to be done are to divide by 2 and subtract 7 from both sides.
$2x + 7 - 7 = 5 - 7$	We must do the "-7" first.
$2x = -2$	
$\frac{\cancel{2}x}{\cancel{2}} = \frac{-2}{2}$	And will finish by dividing both sides by 2.
$x = -1$	

Example 2

$\frac{x}{2} - 3 = 5$	In this example, the two steps that will need to be done are to multiply by 2 and add 3 to both sides.
$\frac{x}{2} - 3 + 3 = 5 + 3$	We must do the "$+3$" first.
$\frac{x}{2} = 8$	
$\cancel{2} \cdot \frac{x}{\cancel{2}} = 2 \cdot 8$	And will finish by multiplying both sides by 2.
$x = 16$	

Solving Linear Equations That Include Parentheses

To solve linear equations that involve parentheses, the first thing we must do is eliminate the parentheses on each side of the equation and then combine like terms. At that point, all we need to do is apply the same techniques we have already been doing.

Example 1

$5(x + 2) - x = 14$	First, distribute the **5** over the parentheses
$5x + 10 - x = 14$	Then, combine the terms $5x$ and $-x$.
$4x + 10 = 14$	To finish, use the techniques from above; subtract 10 from each side followed by dividing both sides by 4.
$4x = 4$	
$x = 1$	

Example 2

$3 + 7(x - 1) = 2$	First, distribute the **7** over the parentheses
$3 + 7x - 7 = 2$	Then, combine the terms *3* and *-7*.
$-4 + 7x = 2$	To finish, use the techniques from above; add 4 to each side followed by dividing both sides by 7.
$7x = 6$	
$x = \frac{6}{7}$	

A Quick Tip

If you get an equation to solve on the placement exam, either solve it directly or check each possible answer using substitution as an alternative strategy. Choose the strategy that gives you the best chance to succeed!

Additional Help

Hippocampus has a video on Solving Linear Equations at the following link:

Solving Multi Step Linear Equations

You can also search YouTube.com for "solving linear equations"

Practice Problems

Solve the following Linear Equations

1. $x + 13 = 4$	2. $x - 7 = 15$	3. $5x = 15$
4. $3x = -14$	5. $2x - 7 = 3$	6. $-5x + 2 = -13$
7. $3(2x + 4) - 5 = 9$	8. $6 - 4(x + 2) + 3x = 1$	

Answers

1. $x = -9$	2. $x = 22$	3. $x = 3$	4. $x = -\frac{14}{3}$
5. $x = 5$	6. $x = 3$	7. $x = \frac{1}{3}$	8. $x = -3$

Polynomials

This section will cover the following topics

Definitions; "Polynomial", "Like Terms" and "Combine Like Terms"
Adding and Subtracting Polynomials
Multiplying Polynomials
Dividing a Polynomial by a Monomial

Definitions; "Polynomial", "Like Terms" and "Combine Like Terms"

A simple way to think of a **Polynomial** is that it is an expression that combines numbers and variables through addition, subtraction and multiplication. It is important to note that division is missing from this list. Additionally, this also implies that the exponent of each variable must be a positive counting number ($1, 2, 3, 4, \ldots$). Here are some examples to illustrate this.

Expressions	Polynomial (Yes or No?)
$x^3 - x^2 + 2x + 7$	**Yes –** only addition/subtract/multiplication are used, and the exponents are positive counting numbers
$2x^{-5} + \frac{7}{y^2} - \sqrt{x}$	**No –** this expression includes the negative exponent "-5", division by y, and a square root – all of which are not allowed for polynomials
$2x^5 + 7y^2$	**Yes –** even though there are multiple variables (x and y) this is still a polynomial

Like Terms are parts of an expression that share the same variable (or variables) and each of those variables has the same exponent.

Terms	Like Terms (Yes or No?)
$2x^2y^3z \text{ and } 7x^2y^3z$	**Yes –** both terms share the same variables with the same exponents; $x^2, y^3, \text{and } z$
$2x^2y^3z \text{ and } 7x^2y^3$	**No –** the first term contains the variable z, however the second terms does not
$2x^2y^3z \text{ and } 7x^4y^3z$	**No –** although the variables are shared by both terms, the exponent of the x variable is "2" in the first term and "4" in the second term

Now that we know what Like Terms are, we can define the phrase, "**Combine Like Terms**". Think apples and oranges; Adding 2 apples to 7 apples to get 9 apples is combining like terms, however we cannot add 2 apples with 7 oranges because they are not "like". Just like two sets of apples can be added (or subtracted), so can like terms.

Examples

$x^2 + x^2 = 2x^2$	$10x^2y + 5x^2y = 15x^2y$	Note that combining like terms is just adding/subtracting the numbers; e.g. $8 - 2 = 6$
$4x^2 - 9x^2 = -5x^2$	$8x^2y^3z - 2x^2y^3z = 6x^2y^3z$	

Adding and Subtracting Polynomials

Adding and Subtracting Polynomials is really just combining like terms with one exception; that exception will be highlighted in examples 2 and 3. Let's look at some examples.

More Examples

Example 1

$3x^2 + 7x - 5 + 5x^3 - 4x^2 + 7x + 11$	First look for terms with the same variables AND exponents (done by color), and then add/subtract as needed. If a term is by itself, like $5x^3$, there is no need to do anything
$3x^2 + 7x - 5 + 5x^3 - 4x^2 + 7x + 11$	
$5x^3 - x^2 + 14x + 6$	

Example 2

$-(2x^2 - 5x + 2) + 4(6x^2 - 9x + 1)$	In this case, there are parentheses that need to be removed before collecting like terms. This is done by distributing the – and the *4*, which means multiplying every term inside the first set of parentheses by – and every term in the second set of parentheses by *4*.
$-2x^2 + 5x - 2 + 24x^2 - 36x + 4$	
$22x^2 - 31x + 2$	Once finished, collect like terms.

A Quick Recap

When adding/subtracting polynomials, remember apples and oranges. If terms have the same variables with the same exponents, they are like terms and can be added/subtracted.

Multiplying Polynomials

Let's start with a quick lesson on multiplying two single term expressions together. We will use the following rule; $x^a \cdot x^b = x^{a+b}$. That is when two expressions with the same base are multiplied (x is the base, a and b are the exponents) we can add the exponents together. Let's look at a few examples

Examples

$2x^2 \cdot 5x^3 = 10x^5$	$12x^2y^5 \cdot 3x^7y = 36x^9y^6$	Multiply the numbers as usual.
$8x^3y^3z^5 \cdot 2x^2y^7z^2 = 16x^5y^{10}z^7$		Then match up the variables (x with x, y with y, etc.) and add their exponents.

Next, let's take a look at multiplying a single term by a polynomial with multiple terms.

Example

$3x^2(2x^3 - 3x^2 + 5x - 4)$	We must first distribute the $3x^2$ to each term inside the parentheses, and then multiply as we did in the last example.
$3x^2 \cdot 2x^3 - 3x^2 \cdot 3x^2 + 3x^2 \cdot 5x - 3x^2 \cdot 4$	
$6x^5 - 9x^4 + 15x^3 - 12x^2$	

To finish the multiplication of polynomials, we will multiply two binomials together using the technique called FOIL. FOIL gives the order in which the terms of each binomial should be multiplied; First – Outside – Inside – Last

Example

First Term · First Term	$(x+3)(x-7) = x \cdot x$
Outside Term · Outside Term	$(x+3)(x-7) = x^2 - 7 \cdot x$
Inside Term · Inside Term	$(x+3)(x-7) = x^2 - 7x + 3 \cdot x$
Last Term · Last Term	$(x+3)(x-7) = x^2 - 7x + 3x - 21$
Finally, combine any like terms	$(x+3)(x-7) = x^2 - 4x - 21$

Dividing a Polynomial by a Monomial

Dividing a Polynomial by a Monomial means dividing a polynomial by a single term. Here are a couple of examples of what that looks like.

Example 1

$\frac{10x^7}{2x^2}$	This is an example of dividing a single term polynomial by a single term. It is best to work with numbers and variables separately.
$\frac{10x^7}{2x^2} = \frac{5x^7}{x^2}$	**10** ÷ **2** = **5**
$\frac{5x^7}{x^2} = 5x^{7-2} = 5x^5$	Then subtract the exponent of x in the denominator from the exponent in the numerator.

Example 2

$\dfrac{10x^5 - 4x^4 + 7x^3 + 2x^2}{2x^2}$	This is an example of dividing a polynomial with multiple terms by a single term. We will again work with numbers and variables separately.
$\dfrac{10x^5}{2x^2} - \dfrac{4x^4}{2x^2} + \dfrac{7x^3}{2x^2} + \dfrac{2x^2}{2x^2}$	Before we do that, we must split the polynomial in the numerator.
$5x^3 - 2x^2 + \dfrac{7}{2}x + 1$	Once that is done, work term by term using the same techniques as was done in example 1.

A Quick Recap

When multiplying/dividing polynomials, remember to work with numbers and variables separately. When working with variables, exponents are added for multiplication and subtracted for division

Practice Problems

Perform the following polynomial arithmetic

1. $2(3x^2 - 8x + 9) - 3(6x^2 - 4x + 1)$	2. $7(5x^3 - x^2 + 4\text{x}) - (2x^2 - 3x + 4)$
3. $2x^2(6x^2 - 4x + 1)$	4. $(2\text{x} + 7)(3\text{x} - 1)$
5. $\frac{9x^4 - x^3 + 3x^2}{3x^2}$	

Answers

1. $-12x^2 - 4x + 15$	2. $35x^3 - 9x^2 + 31\text{x} - 4$
3. $12x^4 - 8x^3 + 2x^2$	4. $6x^2 + 19\text{x} - 7$
5. $3x^2 - \frac{1}{3}x + 1$	

Additional Help

Hippocampus has videos on Polynomials at the following links:

Adding and Subtracting Polynomials; Multiplying Polynomials; Dividing by a Monomial

You can also search YouTube.com for “adding polynomials”, “multiplying polynomials”, or “dividing polynomials”

Factoring Polynomials

This section will cover the following topics

Factoring the Greatest Common Factor
Factoring Trinomials by Trial and Error
Solving Equations by Factoring

Factoring the Greatest Common Factor

The most basic type of factoring for polynomials is to factor out the **Greatest Common Factor (GCF)**. The goal of factoring is to undo multiplication. Let's take a look at what multiplying a single term into a polynomial looks like, and then we will work backwards.

Example of Multiplication of a Polynomial by a Single Term

$\mathbf{3x^2}(2x^3 - 3x^2 + 5x - 4)$	We must first distribute the $\mathbf{3x^2}$ to each term inside the parentheses, and then multiply term by term.
$\mathbf{3x^2} \cdot 2x^3 - \mathbf{3x^2} \cdot 3x^2 + \mathbf{3x^2} \cdot 5x - \mathbf{3x^2} \cdot 4$	
$6x^5 - 9x^4 + 15x^3 - 12x^2$	

Working backwards, let's start with the polynomial $6x^5 - 9x^4 + 15x^3 - 12x^2$. When factoring the GCF deal with the numbers and each variable separately to determine the overall GCF.

Finding the GCF of $6x^5 - 9x^4 + 15x^3 - 12x^2$

GCF of the Coefficients (dealing with the numbers)	6, 9, 15, and 12 are the coefficients All of these numbers are divisible by 1 and 3 only. Always take the highest number, which is this case is 3
GCF of the Variable (dealing with the *x-variable)*	These include x^5, x^4, x^3, and x^2 To find the GCF of variables, take the variable raised to the lowest exponent. In this case, that is x^2
The Overall GCF	Putting the GCF of the numbers and variables together, we get $\text{GCF} = 3x^2$
Factor the GCF Start by factoring $3x^2$ from each term. Then factor $3x^2$ outside parentheses and the remaining terms inside	$6x^5 \quad - \quad 9x^4 \quad + \quad 15x^3 \quad - \quad 12x^2$ $\mathbf{3x^2} \cdot 2x^3 \quad - \quad \mathbf{3x^2} \cdot 3x^2 \quad + \quad \mathbf{3x^2} \cdot 5x \quad - \quad \mathbf{3x^2} \cdot 4$ $\mathbf{3x^2}(2x^3 - 3x^2 + 5x - 4)$

Factoring Trinomials by Trial and Error

Once again, we will start with the idea that factoring will undo multiplication. For trinomials (polynomials with three terms), this means we will be undoing FOIL-ing (see the review on Polynomials for details).

Example of FOIL-ing

First Term · First Term	$(\boldsymbol{x}+3)(\boldsymbol{x}-7)=\boldsymbol{x}\cdot\boldsymbol{x}$
Outside Term · Outside Term	$(\boldsymbol{x}+3)(x\boldsymbol{-7})=x^2\boldsymbol{-7\cdot x}$
Inside Term · Inside Term	$(x\boldsymbol{+3})(\boldsymbol{x}-7)=x^2-7x\boldsymbol{+3\cdot x}$
Last Term · Last Term	$(x\boldsymbol{+3})(x\boldsymbol{-7})=x^2-7x+3x\boldsymbol{-21}$
Finally, combine any like terms	$(x+3)(x-7)=x^2-4x-21$

To work backwards, we will start by considering the possible ways to factor the first term x^2 and the last term -21. We will then write all possible factorizations based on those

Example: Factor $x^2-4x-21$

Possible Factors of First Term	Possible Factors of Last Term	Possible Factorization	Check by FOIL-ing
$\boldsymbol{x^2=x\cdot x}$	$\boldsymbol{-21=-3\cdot 7}$	$(\boldsymbol{x-3})(\boldsymbol{x+7})$	$x^2+4x-21$
$\boldsymbol{x^2=x\cdot x}$	$\boldsymbol{-21=3\cdot -7}$	$(\boldsymbol{x+3})(\boldsymbol{x-7})$	$x^2-4x-21$*
$\boldsymbol{x^2=x\cdot x}$	$\boldsymbol{-21=-1\cdot 21}$	$(\boldsymbol{x-1})(\boldsymbol{x+21})$	$x^2+20x-21$
$\boldsymbol{x^2=x\cdot x}$	$\boldsymbol{-21=1\cdot -21}$	$(\boldsymbol{x+1})(\boldsymbol{x-21})$	$x^2-20x-21$

*Note that we could have stopped at the second row because we found the factorization.

Example 2: Factor $2x^2-5x+3$

Possible Factors of First Term	Possible Factors of Last Term	Possible Factorization	Check by FOIL-ing
$\boldsymbol{2x^2=2x\cdot x}$	$\boldsymbol{3=1\cdot 3}$	$(\boldsymbol{2x+1})(\boldsymbol{x+3})$	$2x^2+7x+3$
		$(\boldsymbol{2x+3})(\boldsymbol{x+1})$	$2x^2+5x+3$
$\boldsymbol{2x^2=2x\cdot x}$	$\boldsymbol{3=-1\cdot -3}$	$(\boldsymbol{2x-1})(\boldsymbol{x-3})$	$2x^2-7x+3$
		$(\boldsymbol{2x-3})(\boldsymbol{x-1})$	$2x^2-5x+3$

A Quick Tip

Factoring can get complicated very quickly, and so can factoring techniques. On the placement exam, keep it simple. These represent the difficulty level you will find on the exam.

Solving Equations by Factoring

A very important point about solving equations by factoring is that one side of the equation must be equal to zero. Once you have that, solving equations by factoring is easy; simply factor and then set each factor equal to zero.

Example 1: $2x^2 - 6x = 0$

Factor out the GCF	$2x(x-3) = 0$	
Set each factor equal to zero	$2x = 0$	$x - 3 = 0$
Solve each equation	$2x = 0$	$x = 3$

Example 2: $3x^2 - 5x = 2$

Write equation with $= 0$	$3x^2 - 5x - 2 = 0$	
Factor	$(3x+1)(x-2) = 0$	
Set each factor equal to zero	$3x + 1 = 0$	$x - 2 = 0$
Solve each equation	$x = -\frac{1}{3}$	$x = 2$

Practice Problems

Factor the following expressions, or factoring to solve the following equations

1. $6x^5 + 9x^4 - 24x^3 + 18x^2$	2. $x^2 - 4x - 32$	3. $3x^2 + 14x - 5$
4. $3x^2 - 4x = 0$	5. $x^2 - 3x - 28 = 0$	6. $2x^2 - 5x = 7$

Answers

1. $3x^2(2x^3 + 3x^2 - 8x + 6)$	2. $(x-8)(x+4)$	3. $(3x-1)(x+5)$
4. $x = 0, \frac{4}{3}$	5. $x = 7, -4$	6. $x = -1, \frac{7}{2}$

Additional Help

Hippocampus has videos on factoring at the following links:

Factoring by GCF; Factoring Trinomials; Solve Equation by Factoring

You can also search YouTube.com for “factoring the GCF”, “factoring trinomials”, or “solving quadratic equations by factoring”

LINES

This section will cover the following topics

Slope and the Slope Formula

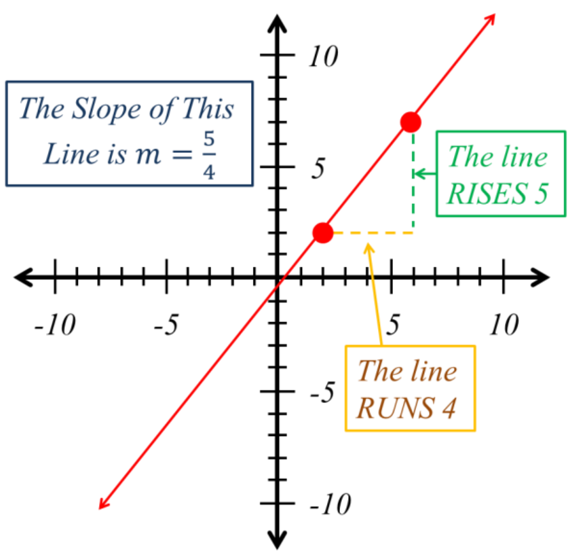
Equation of a Line in Slope-Intercept Form

Graphing Lines

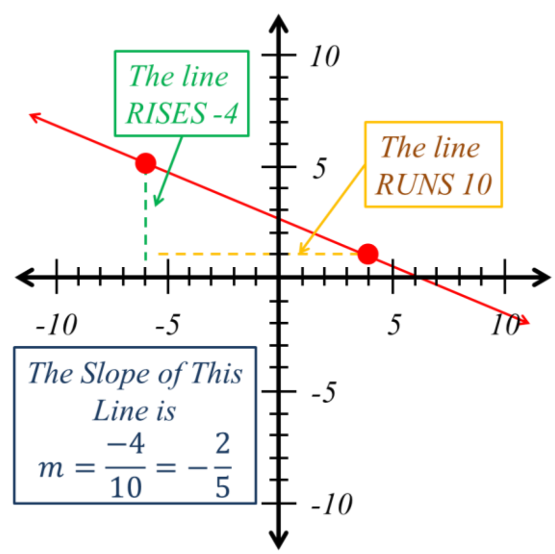
Slope and the Slope Formula

Every line travels in a specific direction. That direction is referred to as the slope of a line, which is often expressed as $m = \frac{RISE}{RUN}$; that is, a measure of how quickly a line rises (or falls) relative to quickly it runs (or travels to the right). The examples below illustrate this.

Example 1



Example 2



An analytical way of determining the slope of a line is through the slope formula, which is

$m = \frac{y_2 - y_1}{x_2 - x_2}$. Here is an example of how the slope formula is used.

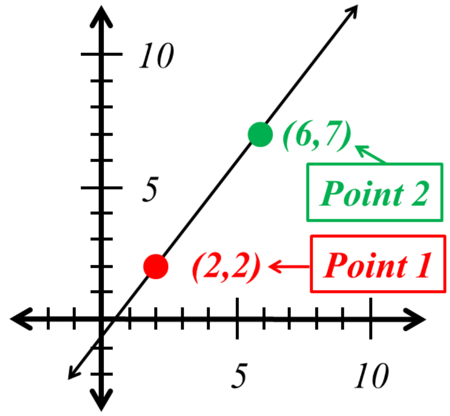
Using the graph on the right, will let

$(x_1, y_1) = (2, 2)$, and

$(x_2, y_2) = (6, 7)$.

From here we do a substitution into the slope formula to get,

$$m = \frac{y_2 - y_1}{x_2 - x_2} = \frac{7-2}{6-2} = \frac{5}{4}$$



Equation of a Line in Slope-Intercept Form

The slope-intercept form of the equation of a line is $y = mx + b$, where m represents the slope and b represents the *y-intercept*. More precisely, the *y-intercept* is the point $(0, b)$. Note that the *y-intercept* occurs when $x = 0$, thus the *y-intercept* is $(0, b)$ no matter what the value of b. Let's look at some examples of identifying the slope and intercept from such equations.

Equation	Slope	y – intercept	Notes
$y = 5x - 2$	$m = 5$	$(0, -2)$	It may be helpful to write $m = \frac{5}{1}$; that is, $RISE = 5$ and $RUN = 1$
$y = -\frac{3}{5}x + \frac{1}{2}$	$m = -\frac{3}{5}$	$\left(0, \frac{1}{2}\right)$	Be sure to include the negative sign with the slope

A Quick Tip

With *x-intercepts* and *y-intercepts*, we already know half of the ordered pair;

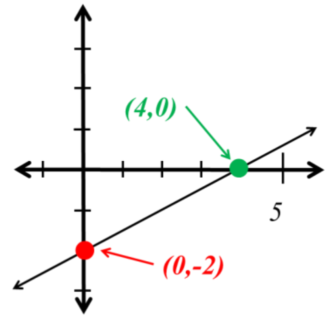
Remember a line crosses the *y-axis* (the *y-intercept*) when $x = 0$

and a line crosses the *x-axis* (the *x-intercept*) when $y = 0$

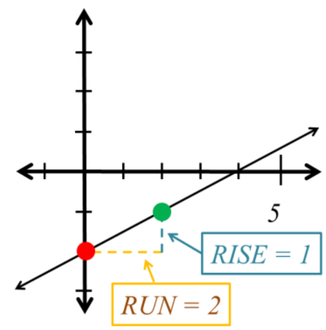
Graphing Lines

Graphing lines starts with a very simple concept. Draw two points and then connect them with a straight line. The only question is how you get the two points. We will take a look at two methods to graph the line $y = \frac{1}{2}x - 2$

Method 1: Determine the x and y-*intercepts*

y-intercept	*x-intercept*	
This one is easy because the equation $y = \frac{1}{2}x - 2$ tells us that the *y-intercept* is the point $(\mathbf{0}, \mathbf{-2})$	We let $y = 0$ $0 = \frac{1}{2}x - 2$ Solving for x gives $x = 4$ The x-intercept is $(\mathbf{4}, \mathbf{0})$	

Method 2: Graph the y-intercept and then use the slope to create a second point.

y-intercept	*slope*	
Again this is easy because the equation $y = \frac{1}{2}x - 2$ tells us that the *y-intercept* is the point $(\mathbf{0}, \mathbf{-2})$	Here we will interpret the slope $m = \frac{1}{2}$ as $\frac{RISE}{RUN}$. From $(\mathbf{0}, \mathbf{-2})$, we will *RISE 1* and *RUN 2*	

Practice Problems

Use the slope formula to find the slope of the line that connects the following points

1. $(-2,7)$ and $(4,1)$
2. $(-1,-3)$ and $(3,6)$

Identify the slope and y-intercept given the following equations

3. $y = -3x + 4$
4. $y = \frac{2}{3}x - \frac{5}{7}$

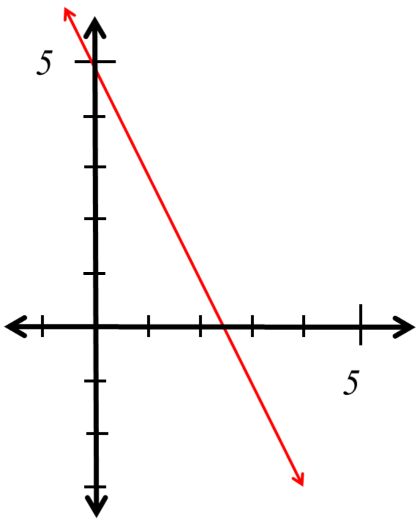
Graph the following lines

5. $y = -2x + 5$
6. $y = \frac{3}{4}x - 2$

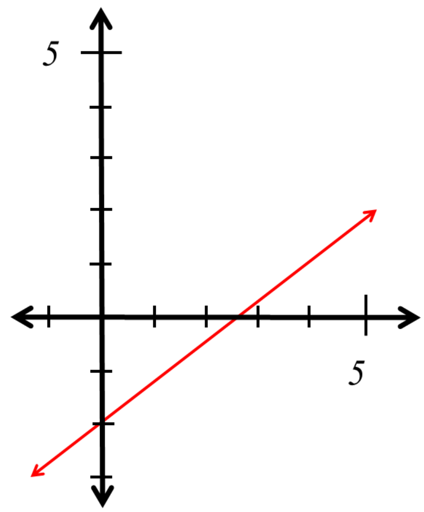
Answers

1. $m = -1$	2. $m = \frac{9}{4}$	3. $m = -3, (0, 4)$	4. $m = \frac{2}{3}, (0, -\frac{5}{7})$

5.



6.



Additional Help

Search YouTube.com for "slope of a line", "slope formula", or "graphing a line"